Contents

0.1. Measurements

von Neumann measurements: $\sum_i P_i = \mathbb{I}$, $P_i P_j = \delta_{ij} P_i$. Then when measuring ρ_A, it collapses to $\frac{1}{\mathrm{tr}(P_i \rho_A)} P_i \rho_A P_i$. If we measure system C on the state $U_{AC}(|0\rangle\langle 0| \otimes \rho_A)U_{AC}^\dagger$ gives $\mathrm{tr}_C\Big(\Big(P_i^{(C)} \otimes \mathbb{I}\Big) U_{AC}(|0\rangle\langle 0| \otimes \rho_A)U_{AC}^\dagger \Big(P_i^{(c)} \otimes \mathbb{I}\Big)\Big)$

Let $A_0 = \sqrt{\mathbb{I} - \mathrm{d}t \sum_i L_i^\dagger L_i}$, $\{L_i\}$ are Limdblod operators, $A_i = \sqrt{\mathrm{d}t} L_i$. This gives

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = i[H, \rho] + \sum_i L_i \rho L_i^\dagger - \frac{1}{2} \sum_i \Big(L_i^\dagger L_i \rho + \rho L_i^\dagger L_i\Big).$$

Ky-Fan principle for Hermitian matrices: $\lambda_1 = \max_{P_1} \mathrm{tr}(P_1 \rho) = \max_{|\psi\rangle} \langle \psi | \rho | \psi \rangle$, $\lambda_1 + \lambda_2 = \max_{P_2} \mathrm{tr}(P_2 \rho)$, $\lambda_1 + \lambda_2 + \lambda_3 = \max_{P_3} \mathrm{tr}(P_3 \rho)$. P_i are projectors.

Theorem 0.1 (Quantum Steering) Let $|\psi\rangle$ be a pure state in $\mathbb{H} = \mathbb{H}_A \otimes \mathbb{H}_B$ and let $\rho_B = \mathrm{tr}_A(|\psi\rangle\langle\psi|)$. A POVM measurement on system A can produce the ensemble $\{(p_i, \rho_i) : i \in [M]\}$ at system B iff $\rho_B = \sum_{i=1}^M p_i \rho_i$.

Remark 0.2 The Quantum Steering theorem is also known as the Hughston, Jozsa, Wootters theorem.

Definition 0.3 An **entanglement monotone** is a function on the set of quantum states in $\mathbb{H}_A \otimes \mathbb{H}_B$ which does not increase, on average, under local transformations on $\mathbb{H}_A$ and $\mathbb{H}_B$. In particular, it is invariant under local unitary operations.

Theorem 0.4 (Vidal) A function of a bipartite pure state is an entanglement monotone iff it is a concave unitarily invariant function of its local density matrix.

Example 0.5 Let $\mathbb{H} = \mathbb{H}_A \otimes \mathbb{H}_B$ with $n = \min\{\dim \mathbb{H}_A, \dim \mathbb{H}_B\}$. A family of entanglement monotones on $\mathbb{H}$ is given by

$$\mu_m(|\psi\rangle) = -\sum_{i=1}^m \lambda_i,$$

for each $m \in [n]$, where $\lambda_1, \ldots, \lambda_n$ are the Schmidt coefficients of $|\psi\rangle$ in decreasing order.

Definition 0.6 Let $x, y \in \mathbb{R}^n$, and let $x^{(i)}$ denote the i-th largest element of x. We say x **weakly majorises** y, written $y \underset{w}{\prec} x$, if

$$\sum_{i=1}^m y^{(i)} \leq \sum_{i=1}^m x^{(i)} \quad \forall m \in [n].$$

x **majorises** y if it weakly majorises y and $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i$.

Theorem 0.7 The probabilistic transformation $|\psi\rangle \mapsto \{(p_i, |\psi_i\rangle) : i \in [M]\}$ can be accomplished using LOCC iff

$$\lambda(|\psi\rangle) \prec \sum_{i=1}^M p_i \lambda(|\psi_i\rangle),$$

where $\lambda(|\varphi\rangle)$ denotes the vector of Schmidt coefficients of $|\varphi\rangle$.

Theorem 0.8 (Bennett)

Theorem 0.9 Fundamental theorem of MPS: $|\psi(A)\rangle = |\psi(B)\rangle$ iff $\exists \varphi, X$ such that $B^i = e^{i\varphi} X A^i X^{-1}$.

Smith normal form: if matrix M has integer entries, can write $M = U\Sigma V^T$, where $\det(U), \det(V) = \pm 1$, U, V have integer entries, Σ is diagonal with entries